On the Inner Radius of Nodal Domains

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Abstract

Let (M, g) be a closed compact smooth Riemannian Manifold of dimension n. Let Δ be the Laplace-Beltrami Operator on M. We consider the eigenvalue equation $\Delta\varphi_\lambda = \lambda\varphi_\lambda$. The λ-*nodal set* is the set $\{\varphi_\lambda = 0\}$, and any connected component of the complement $\{\varphi_\lambda \neq 0\}$ is called a λ-*nodal domain*.

Faber-Krahn Inequality shows that

$$\text{Vol}(\text{a } \lambda\text{-nodal domain}) \geq (C/\sqrt{\lambda})^n .$$

We prove that in dimension two one can in fact inscribe a ball of radius $C/\sqrt{\lambda}$ in any λ-nodal domain, i.e.,

$$\text{Inrad}(\text{a } \lambda\text{-nodal domain}) \geq C/\sqrt{\lambda} .$$

In dimension $n \geq 3$, we show

$$\text{Inrad}(\text{a } \lambda\text{-nodal domain}) \geq (C/\sqrt{\lambda})^{n-1} .$$

We show that this problem is closely related to a connection between the growth of harmonic functions and their zeroes.

References

[1] *Local Asymmetry and the Inner Radius of Nodal Domains*, to appear in Comm. Partial Differential Equations, arXiv:math/0703663.