August 02, 2015

Dear Math Concentrators,

Welcome back!

In your Communication Packet via online registration you will find several pamphlets prepared by the Department. If you haven't read them before, we urge you to do so: they contain useful information on selecting math courses, applying to graduate schools, getting part-time and summer jobs in mathematics, selecting senior thesis topics and advisors, etc. In particular, they describe the Math Expository requirement that students are required to comply with. Even if not all of these topics seem relevant for you, please browse through the pamphlets anyway – you may be surprised at what you learn.

There will be three tutorials offered this year: one in the Fall and two in the Spring.

Fall 15:

- *Category Theory*, taught by Danny Shi.

Spring 16:

- *Morse Theory*, taught by Boyu Zhang
- *Patitions, Young Diagrams and Beyond*, taught by Konstantin Matveev.

The description of the tutorials will be posted on the undergraduate bulletin boards (one opposite rm. 320, the other opposite rm. 503). They will also be e-mailed to all math concentrators on our mailing list and posted on the Math Department website http://www.math.harvard.edu.

We would also like to remind you of some upcoming events:

Tuesday, 9/1, 5:30 pm. The first meeting of the Math Club at Mather House.

Wednesday, 9/2, 4:00 pm. Organizational sign-up meeting for the Fall Tutorials in room 530.

Friday, 9/4, 4:00 pm. Welcome Party in the Austin and Chilton McDonnell Common Room, fourth floor, Science Center.

Please check the Undergraduate bulletin boards periodically for more announcements.

Have a good semester,

Jacob Lurie
Director of Undergraduate Studies

Beyond Math 1:
Which math course is for you?
(2015-2016)

If you have completed the Math 1a/1b sequence at Harvard or if you have had the equivalent material elsewhere, you may be wondering which course is for you. The mathematics department provides a variety of options which you should consider based on your academic interests and your background. With exceedingly rare exceptions, students in your position are advised to take one (or more) of Math 18, 19a, 19b, 21a, 21b, 23a, 25a, 55a, or 101. (The School of Engineering and Applied Sciences also offers Applied Math 21a,b which covers selected topics from Math 21.) This pamphlet describes the Mathematics Department's offerings and should help you decide which course is for you.

Math 18 focuses on concepts and techniques of multivariable calculus most useful to those studying the social sciences, particularly economics: functions of several variables: partial derivatives; directional derivatives and the gradient: constrained and unconstrained optimization, including the method of Lagrange multipliers. Covers linear and polynomial approximation and integrals for single variable and multivariable functions: modeling with derivatives.

Math 19a and 19b are courses that are designed for students concentrating in the life sciences. (These courses are recommended over Math 21a,b by the various life science concentrations.) Math 19a is taught in the fall and repeated in the spring; it focuses on differential equations, related techniques and modeling. Math 19b teaches linear algebra, probability and statistics; it is offered only in the spring. Both courses focus on applications and examples from the life sciences. If you passed Mathematics 1b (or have the permission of the instructor), you can take Mathematics 19a,b.

Math 21a,b is the standard second-year calculus and linear algebra sequence. It is normally taken by those students who intend to concentrate in the physical sciences or mathematics and who have had a solid first year calculus course. Math 21 emphasizes computational techniques and applications. It seeks to develop tools and intuition rather than spend time proving the results used. Math 21 is given in semester-long halves which may be taken in either order or concurrently. Math 101 can be taken concurrently with either Math 21a or 21b. The material in Math 21a/b is presented, where feasible, in correlation with Physics 15/16.

Math 21a covers multivariable calculus, while Math 21b is a one-semester introduction to linear algebra and differential equations. First-year students who had an equivalent of Math 21a in high school often take this course in the fall of their freshman year. The students with such background who intend to major in math or theoretical physics should also look into Math 23, Math 25, or Math 55. Those who are considering a concentration in mathematics may want to take Math 101 concurrently with either Math 21a or b.

(over)

Math 101 is a one-semester introduction to the three main branches of modern mathematics (algebra, analysis, and geometry) and to the methodology used in higher mathematics. It has no official prerequisites. In this course students learn to write rigorous proofs and encounter fundamental concepts which are further developed in other 100-level courses. Math 101 is intended both for those who wish to concentrate in mathematics and for those in other fields (related or not) who have an interest in learning what higher math is all about. Students often take it concurrently with or right after Math 21. Those who are taking or have taken Math 23, 25, or 55 should not take 101. In 2015-16, Math 101 will be given in the spring, but not in the fall semester.

Math 23 is an advanced version of the 21 sequence designed for students with strong math interest. This course develops theories of functions of several variables and linear algebra. Students in this course will learn to write rigorous proofs and encounter some of the beauty and elegance of modern mathematics. Math 23 offers a theoretical understanding of the mathematical concepts which are taught in Math 21. Please note that Math 23 *may not* correlate with the Physics 15/16 sequence. Also note that all 100 level Math courses which accept Math 25 or 55 as a prerequisite also accept Math 23. Math 23 should not require an unusual out of class time commitment.

Math 25 and 55 are both full-year advanced courses designed for students with a very strong interest in theoretical mathematics. Each covers multivariable calculus, linear algebra, and some additional topics from a rigorous and advanced point of view. The students in these courses are frequently committed to concentrating in mathematics and are asked to put in extensive work outside the classroom. Many have had more than one year of college mathematics while in high school or have participated in various summer math programs. However, it is not necessary to have had multivariable calculus before taking 25 or 55. Although the syllabus of Math 25 is similar to that of Math 23, students will usually have had more preparation in math.

Math 55 is a faster paced course and covers topics more deeply. It is designed for students who arrive at Harvard with an extensive background in college level math. Math 25 and 55 differ from Math 23 in the level of outside work required: homework assignments in Math 25 and 55 are typically very time consuming. Math 23, 25 or 55 all provide an excellent foundation for further study of mathematics.

Skipping Math 25 and 55: Every year a few freshmen want to skip the Math twenty/fifty level all together and start with a 100- or 200- level course. The Department, based on many years of experience, *strongly discourages* this. You may learn more advanced material in higher level courses, but never at the same speed and intensity as in Math 25 or 55. Moreover, you are learning more than just a body of mathematics in these courses. You are also learning how to 'be' a research mathematician (as opposed to one who only does well in Math courses). If, in spite of this warning, you think that taking a higher level course as a freshman would best serve your needs, you should speak to the Director of Undergraduate Studies of the Mathematics Department, Professor Jacob Lurie (lurie@math).

Preparing for the Mathematics Concentration

(2015-2016)

This document is for you if you are considering becoming a concentrator in Mathematics and want a little more information. If you are in the Spring semester of your Freshman year, or are beginning your Sophomore year, and are interested in Mathematics, then this document will help to answer the questions "What does the Mathematics concentration have to offer?", "What are the key courses in the Mathematics curriculum?", "What should I do now, to prepare for the Mathematics concentration", and "Where can I can I get more information and advice?"

But this is a "quick-start" document. For more in-depth information, please look at the other pamphlets that are available either from the Undergraduate Studies Coordinator (Cindy Jimenez, room 334 in the Science Center), or from the undergraduate section of the Department's web site at http://www.math.harvard.edu/. In particular, you should look at:

- the pamphlet *Courses in Mathematics*, which describes some of the courses and advises you on how to structure a good program; and
- the pamphlet *Concentration in Mathematics*, which gives fuller answers to some of the general questions and gives some specifics about the concentration requirements and the departments facilities.

Why Mathematics?

Concentration in mathematics is an excellent preparation for a career in academia or in industry. Because physics, chemistry, computer science, economics, and even social sciences rely heavily on mathematical methods, a math concentration can provide an invaluable background for many different fields of endeavor. Concentrators who do not choose to continue in mathematics have often gone on to graduate work in other academic subjects, to actuarial and computer science careers, or to professional training in law, business or medicine. A math concentration is very flexible and has a reasonably small number of requirements, so there are ample opportunities to take electives in related and unrelated fields.

What are the concentration requirements?

In brief, the basic requirements are twelve regular, letter-graded half-courses, of which eight must be courses labeled as Mathematics and four may be math courses or courses drawn from an approved list of related subjects, details of which are published in the

Handbook for Students. Concentrators must include at least one half-course at the l00-level in each of three areas: analysis, algebra and geometry. There is an expository requirement that must be completed by the end of the Junior year, and candidates for Honors are required to write a senior thesis. More details are given in the pamphlet *Concentration in Mathematics*, and the exact description is contained in the *Handbook for Students*.

We also have students taking joint concentrations such as Physics+Math, Math+CS, Philosophy+Math and others. See the pamphlet *Concentration in Mathematics* for more details.

What courses should I be taking as a Sophomore?

If you wish to keep the door open to concentrating in Mathematics, you will want to take some Mathematics in the Fall of your Sophomore year. Here is some advice. It should be read alongside the department's pamphlet "Courses in Mathematics".

- **If you began with Math 1a or 1b** and have not yet completed the Math 21a,b sequence, then that is probably your best course. At the same time, you should consider taking the Fall course Math 101, which offers a taste of higher mathematics and will familiarize you with proofs; it can be taken concurrently with Math 21.

- **If you have completed Math 21a,b**, then you may want to consider Math 101 (just mentioned), Math 112 (Real Analysis, a spring course) or Math 121 (Linear Algebra) in the Fall. These courses teach proof-writing skills, and are designed for students with a Math 21 background. Other courses that are accessible with at this level are Math 152 (Combinatorics), Math 154 (Probability) and Math 130 (Classical Geometry).

- **If you have completed Math 23a,b or Math 25,a,b**, then you should certainly look at Math 122, which is the first of two courses on Abstract Algebra. Many further courses in Mathematics depend on it. If you have done well in your Freshman math courses and can take on more, you might consider also Math 131 (Topology) or perhaps Math 114 (Measure, Integration and Banach Spaces) in the Fall. For many students, however, Math 122 by itself will be more appropriate.

- **If you took Math 55a,b** you should look at Math 114 and Math 131 in the Fall, and perhaps Math 123 and 132 in the Spring.

Look at the pamphlet *Courses in Mathematics*, as well as the course catalog, for more information on the many other courses that are available and how to structure a complete

program. If you are not yet sure that Mathematics is the concentration for you, then bear in mind that courses such as Mathematics 122, 121 and 112 provide tools that will be useful in many other disciplines.

How can I keep in touch?

Advising Fortnight During Advising Fortnight there will be several opportunities to meet and talk to members of the department. In particular, there will be an "open house" with refreshments in the Austine & Chilton McDonnell Common Room (fourth floor of the Science Center) and there will be walk-in office hours during the first week.

Communication and e-mail Send an e-mail with your name, your fas.harvard.edu e-mail address and your year at Harvard to Cindy Jimenez (cindy@math), and ask to be put on our mailing list. The mailing list provides concentrators and potential concentrators with information about upcoming events.

Math Club and Math Table The department has an undergraduate Math Club that meets every Tuesday at 5:30 pm in the Mather House Dining Rooms A & B. It is an opportunity to hear talks over dinner, by undergraduates and faculty. See the pamphlet *Concentration in Mathematics* for more details. If you would like to give a talk at Math Table, please contact Professor Noam Elkies (elkies@math).

Concentrators' Party At the beginning of the Fall semester, the Mathematics Department holds a party for math concentrators in the Austine & Chilton McDonnell Common Room on the 4th floor of the Science Center. Please come and meet other math majors and faculty members. Refreshments provided.

Advising All Mathematics concentrators are assigned a faculty member as an advisor, to help with course selection and other matters. In addition, all Juniors come in for a scheduled 20 minute advising session with two faculty members, to track progress and discuss future steps. The Director of Undergraduate Studies is also available for advising.

For more information Visit the undergraduate pages of the department's web site for copies of pamphlets covering other topics, including advice about Freshman math courses, senior theses, honors requirements, graduate school and more. The pamphlets are also available from the Undergraduate Studies Coordinator, Cindy Jimenez (cindy@math) in room 334 of the Science Center. Further questions can be addressed

to Cindy Jimenez or to the Director of Undergraduate Studies, Professor Jacob Lurie, either at office hours or by email at lurie@math.harvard.edu.

Concentration in Mathematics

(2015-2016)

What Can You Do With Math?

Concentration in mathematics is an excellent preparation for a career in either pure or applied mathematics, in academia or in industry. Because physics, chemistry, computer science, economics, and even social sciences rely heavily on mathematical methods, a math concentration can provide an invaluable background for many different careers. Concentrators who do not choose to continue in mathematics have often gone on to graduate work in other academic subjects, to actuarial and computer science careers, or to professional training in law, business or medicine. A math concentration is very flexible and has a reasonably small number of requirements, so there are ample opportunities to take electives in related and unrelated fields.

Math concentrators who would like to earn teaching certification to teach in Massachusetts public schools after graduation may want to look into the Undergraduate Teachers Education Program (UTEP). More information about UTEP is available from their web site at http://www.fas.harvard.edu/∼ utep/. The Mathematics Department offers a "Mathematics and Teaching" option for those students concurrently enrolled in UTEP. Students choosing this option need to fulfill slightly different course requirements to receive the B.A. in Mathematics.

Concentration Requirements

Concentration in mathematics requires a minimum of twelve regular, letter-graded, half-courses of which eight must be courses labeled as Mathematics and four may be math courses or ones in related subjects. (Note that a specially certified Freshman Seminar can be substituted for one of the eight Mathematics courses.) Concentrators must attain at least one half-course at the 100-level in each of the three areas: analysis, algebra, and geometry. Students choosing the "Mathematics and Teaching" option haveslightly different course requirements.

In addition to the course requirements, all students are required to satisfy the Math Expository requirement (see the section below). The exact description of the requirements and the list of related courses permitted to count for course requirement are printed in the *Handbook for Students*.

Expository Requirement

Each concentrator is required to submit a 5-page expository paper in Mathematics. The paper should be a coherent, correct and original exposition in a subject of pure

or applied mathematics. The paper should be written during the sophomore or junior year under the supervision of a professor or tutor in a tutorial (Math 99r) or a 100- or a 200-level course that the student is contemporaneously enrolled in. The paper has to be accepted by both that professor or tutor, as well as the Director of Undergraduate Studies. The student should be prepared to discuss the contents of the paper with the professor or the Director of Undergraduate Studies.

Ordinarily students enrolled in a tutorial (Math 99r) automatically satisfy the expository requirement as part of the structure of the tutorial.

The expository requirement has to be met before the last day of the spring reading period of the junior year. Extensions may only be granted by the Director of Undergraduate Studies.

Honors

A candidate for Honors must, in addition to satisfying the above mentioned requirements, submit a senior thesis. The thesis may be on any topic in pure or applied mathematics not directly covered in a student's course work. It need not be an original piece of mathematical research, but should be an original exposition of material culled from several sources. The department strongly urges concentrators to write a senior thesis; this experience tends to provide a much better glimpse of mathematical research and graduate work than taking courses alone. If you think you will write a thesis, be on the lookout for a thesis topic in your junior year. More about senior theses can be found in the pamphlet "Honors in Mathematics", available from the Undergraduate Program Coordinator Cindy Jimenez, room 334. Also, feel free to talk with the Director of Undergraduate Studies, Jacob Lurie, if you need help deciding on either doing a thesis or finding a topic or and advisor.

Transfer Credits

The Department encourages students to take the most advanced courses for which they are qualified. Nevertheless, students who enter as freshmen or advanced standing sophomores will not ordinarily be permitted to count courses taken elsewhere toward the twelve-course requirement. Transfer students wishing to concentrate in mathematics should consult the Head Tutor who will review their transcripts and arrange their concentration requirements.

Changing Concentrations to Math

The Department welcomes students who want to change their concentration to Mathematics as long as it is plausible that they can fulfill the requirements within the time

remaining. Students considering Mathematics may also wish to consider Applied Mathematics, Computer Science, any physical science, or Statistics.

Joint Concentrations

Joint concentrations with other departments are possible to arrange. Common joint concentrations are with Computer Science, Physics, Philosophy and Statistics. Joint concentrations are honors only. If Math is primary, the student must fulfill all of the requirements for Honors in Math (including a senior thesis in Math) plus whatever other requirements are called for by the secondary department. (Consult the secondary department for this information.) If Math is secondary, the Math requirements are as follows: five, letter-graded semester courses in Mathematics with a grade of C− or higher, with at least one course numbered in 110-119, one in 120-129 and one in 130-139. The student must fulfill the honors requirements of the primary field only if Math is secondary. In either case, the final determination of honors is made after consultations between the primary and secondary field.

The AB-AM degree program

The Department offers the AB-AM degree, which allows students who are Mathematics concentrators to obtain a Masters degree (AM) in Mathematics, in addition to their Bachelors degree (AB) during their four years at Harvard. Any undergraduate who wishes to apply for this degree must file an application form for the graduate program in mathematics, just as any other student files for graduate work at Harvard. Only students with Advanced Standing are eligible to apply for this four-year program, and the Department will only accept candidates who have demonstrated sufficient mastery of undergraduate material. If you are considering going on to graduate school in mathematics, bear in mind that the Masters degree will offer you no advantage. The program requires a large number of additional courses in mathematics. In nearly all cases, the Department recommends that students should instead take advantage of the many other academic opportunities that the University offers. Further details of the requirements are available at the GSAS website, http://www.gsas.harvard.edu/, under *Programs of Study*, and from the Graduate Studies Coordinator.

Reading and Tutorials Courses

There are two types of Reading Courses available. Math 60r is designed to give more time for thesis work to senior honors candidates; Math 91r is for students who want to learn a particular topic not covered in a regular course or tutorial. Please read the relevant section of the "Courses in Mathematics" pamphlet to learn more about the requirements and modalities of these courses.

Although tutorials (Math 99r) are not required, students are encouraged to take a tutorial in the course of their studies. Students can take as many tutorials as they want, but only one tutorial may count for the concentration requirements.

Generally one or two tutorials are offered every semester. Typically, tutorials are directed by graduate students, and have four to eight students in them. They tend to be less formal and structured than regular courses, yet require more involvement on the part of the students, who have to make presentations and write papers. Very frequently a topic studied in a tutorial leads naturally to a senior thesis, and the paper submitted for the tutorial will generally satisfy the Math expository requirement.

A description of offered tutorials is placed into concentrators' registration envelopes in September, and another is e-mailed in January. Tutorial descriptions also appear during the first week of each semester on the undergraduate bulletin boards (one opposite room 320, and one opposite room 503 in the Math Department) and are posted at Math Department's website http://www.math.harvard.edu/. Often, tutorials are previewed at Math Table meetings (For more information on Math Tables, see below). A special organizational meeting for tutorials is held the first Wednesday of the fall semester. The spring semester tutorials are organized in the first week of that semester; see the undergraduate bulletin boards for announcements.

Questions regarding tutorials may be addressed to the Director of Undergraduate Studies, Jacob Lurie (lurie@math), or the Undergraduate Program Coordinator, Cindy Jimenez (cindy@math).

Cross-registration at M.I.T.

Students may cross-register to take a course at M.I.T. It is a particularly useful option for students interested in logic and combinatorics. For more information please see the "Courses in Mathematics" pamphlet. You must get *advance* permission from the Director of Undergraduate Studies, Jacob Lurie (lurie@math), to have M.I.T. courses count for concentration credit.

Concentration Advisor

The department assigns all students a faculty member as their concentration advisor. If you prefer to change your assignment, please talk to Cindy Jimenez in room 334 (cindy@math, tel. 495-9116). Your advisor can help you plan your course-work, and will sign your study card. How much contact you have with your advisor and how helpful he or she is will depend almost entirely on your initiative. Feel welcome to drop by during his or her office hours or during our 4 pm teas (see below), or to invite your advisor to lunch at your House (students can invite a faculty member for any meal at their House, courtesy of Harvard – ask the checker in your House's dining hall for the

form). Your advisor can help you plan your courses, choose a thesis topic, a thesis advisor, learn about mathematical research, and apply to grad school.

To insure a minimum of advising, the department has every junior come in for a scheduled 20 minute advising session with two faculty members in the spring semester; post-graduate options and senior theses are often discussed. Juniors will receive a letter by e-mail in the spring. If you do not receive an appointment letter, please contact Cindy Jimenez (cindy@math).

Math Competitions

Each year a large number of Harvard undergraduates compete in the William Lowell Putnam Mathematical Competition. It is open only to regularly enrolled undergraduates in colleges and universities of the United States and Canada. This is a competitive examination given in two three-hour sessions in early December. There are substantial prizes for both individuals and teams, and Harvard students have done extremely well in the recent past. Sign up to participate in the Putnam competition in late September on the bulletin board opposite room 323. Harvard students have also done well in the MAA-SIAM Mathematical Modeling Contest that occurs in early February.

Math Club and Math Table

The Math Department has an undergraduate math club. The club represents the interests and the views of Math concentrators.

The club also sponsors the Math Table, which meets every Tuesday at 5:30 pm at Mather House Dining Rooms A & B over dinner to hear talks by faculty and undergraduates. Anyone, at any level of math literacy, is welcome to come. The talks at the Math Table generally cover topics which fall outside the scope of regular courses. The talks serve three purposes: to introduce undergraduates to various fields of mathematics, to suggest topics for senior thesis research, and to provide a forum for undergraduates to deliver talks on mathematical subjects. The Rogers Prizes are awarded each year for the two best talks delivered by undergraduates. Announcements about the Math Table talks are posted on the undergraduate bulletin boards (near rm. 320, rm. 503) and e-mailed to the concentrators' e-mail list.

If you would like to give a talk at the Math Table, please contact the Math Table coordinator, Professor Noam Elkies (elkies@math).

Communication and E-Mail

To facilitate communication with its undergraduates, the department urges all mathematics concentrators (potential concentrators and those just interested in math) to forward their e-mail address, together with their name and year at Harvard to Cindy

Jimenez (cindy@math). Information about events of concern to math concentrators (for example, job opportunities, math table talks, tutorials, etc.) will be transmitted as it becomes available. Additional information is also available on the Math Department website.

Term-Time and Summer Jobs, Study Abroad

Course Assisting

The Math Department hires undergraduates as CA's (i.e. course assistants). A CA grades homework assignments, leads a section for an hour a week, has office hours, writes solution sets, and attends the lectures of his or her class. For more information on CA positions for the calculus courses (at the level of Math 21 and below), as well as the 100-level courses, go to http://www.math.harvard.edu/jobs/. Most hiring for these positions takes place in the spring.

Science Advising

To help with Science Advising in the fall contact Professor Jacob Lurie (lurie@math.harvard.edu).

Study Abroad

The Office of Career Services can help you arrange to spend a semester or a year studying abroad. Math concentrators have taken advantage in the past of a well-regarded "Budapest Semesters in Mathematics" program in Hungary. The program allows American students to spend one or two semesters studying combinatorics, algebra, and discrete math in small groups with senior Hungarian mathematicians. All classes are taught in English. Applications for the Fall semester are due at the end of April, for Spring in mid October. For more information you can send e-mail to Paul Humke (humke@stolaf.edu) or visit the website http://www.stolaf.edu/depts/math/budapest.

Summer Research

There are opportunities to do pure or applied math during the summer. Browse through the undergraduate bulletin boards (near rm. 320, rm. 503) periodically during the spring semester.

National Science Foundation sponsors a number of Research Experiences for Undergraduates (REU) programs in mathematics every year. You may wind up learning something interesting, *and* making money!

The Weizmann Institute of Science in Rehovot, Israel has an annual summer research program in science and mathematics for undergraduates. They match students with

researchers and provide a modest stipend. The program is conducted in English, and the applications are usually due in late December. More information is available from their website at http://www.weizmann.ac.il/acadsec/kkiss.html.

Large companies, such as IBM or Bell Labs, often hire undergraduates to do research (usually somewhat applied math) during the summer. If you are interested, write directly to the company very early (December is not too early) and look for advertisements posted on the Division of Engineering and Applied Sciences' bulletin boards.

Those who register their e-mail accounts with the department (see "Communications" above) will periodically receive information about some of these opportunities. More announcements will appear on the bulletin boards. You should also regularly check with the Office of Career Services.

Activities and Resources

Concentrators' Party

At the beginning of the fall semester Math Department holds a party for Math concentrators in the Austine & Chilton McDonnell Common Room on the 4th floor of the Science Center. Please come and meet other math majors and faculty members. Refreshments provided.

Access to the Library and the Department

The Birkhoff Mathematics Library on the third floor of Science Center, holds many mathematical books and journals. All texts used in math courses during the semester are held on reserve there. It is a noncirculating library, so its resources may be used only on the premises. The library is open Monday through Friday 9 to 5 pm.

All Math majors may request the card access to the Math Department from the Main Office in room 325.

The Austine & Chilton McDonnell Common Room

The Austine & Chilton McDonnell Common Room, located on the fourth floor of the Science Center, is open to all concentrators and friends of the Mathematics Department. Tea is served Monday through Wednesday (and occasionally on Thursday) at 4 pm. Cheese, bread, and juice is offered at 4:15 or so on Fridays. Harvard Math Department prides itself on its informal atmosphere. Come to these events regularly, and get to know the faculty and graduate students.

For More Information:

Information about tutorials, jobs, graduate schools, fellowships, and other matters is posted on the Undergraduate bulletin board opposite room 320. You can get the pamphlets "Honors in Mathematics", "Beyond Math 1: Which Math Course is for You?", "Mathematical Sciences at Harvard", "Courses in Mathematics", and "Graduate Schools and Fellowships in Mathematics" from the Undergraduate Program Coordinator Cindy Jimenez (cindy@math) in room 334. These pamphlets, as well as some other information is available from the Math Department homepage at http://www.math.harvard.edu/.

For other information consult the Director of Undergraduate Studies, Jacob Lurie, room 514 (5-9493), (lurie@math).

Courses in Mathematics

(2015-2016)

This document gives a brief description of the various courses in calculus and some of the intermediate level courses in mathematics. It provides advice and pointers for planning your course selections. If you are a Mathematics Concentrator, or are considering entering the Mathematics Concentration, and if you are seeking some overview of the courses and how they fit together, then this document is for you. However, the guidelines presented below are exactly that: guidelines. Keep them in mind when you are deciding how to structure your program, but be sure to talk to your advisor in the Mathematics Department or to the Director of Undergraduate Studies before you turn in your study card each semester.

1 Calculus

Math 1a/b is the standard first-year calculus sequence. If you are thinking about majoring in math and have not taken calculus before, take Math 1 as soon as possible! If you have had a year of calculus in high school, and if you have passed the Advanced Placement examination in BC Calculus with a score of 4 or better, then you may be advised to begin with Math 21 a/b, the second-year calculus sequence.

If you scored a 5 on the BC Calculus exam and if you are advised to take Math 21 a/b, then you may wish to consider taking Math 23 or Math 25 or 55 instead of Math 21. Be warned: Math 23, 25 and 55 are intense but very rewarding courses, and both 25 and 55 require extensive work outside the classroom. To succeed in the latter two, you must be very committed to mathematics from the start.

Regardless of which calculus course you take, keep in mind that it is important to absorb ideas thoroughly. It's a bad idea to push yourself too far too fast.

For more guidance on choosing your first math course at Harvard please read the pamphlet "Beyond Math 1: Which math course is for you?", which you can obtain from Cindy Jimenez, the Undergraduate Program Coordinator (room 334), or from the undergraduate section of the Department's web site.

2 How to structure a good program

No single program is ideal for all math concentrators. You should design your curriculum based on your background, interests, and future plans. You are strongly urged to consult with your academic advisor or with the Director of Undergraduate Studies in deciding which courses are best suited for you. Do not plan to meet with your advisor on the

day study cards are due, since advisors usually don't have more than a few minutes to spend with each student that day. Make an appointment with your advisor well before study cards are due. You should allot about half an hour, so you can discuss your plan of study in depth.

Learning to write proofs

Math 23, 25, 55, 101, 112, and 121 are six courses in which you learn to write proofs, meeting (often for the first time) a style of mathematics in which definitions and proofs become part of the language. Students are generally advised not to take any upper-level math courses before completing (or, at least, taking concurrently) one of these.

- Math 101 serves three main goals. It lets a student sample the three major areas of mathematics: analysis, algebra, and topology/geometry; it introduces the notions of rigor and proof; and it lets the student have some fun doing mathematics. If you are considering concentrating in Mathematics but are not sure that you are ready to take Math 23, 25 or 55, or if you simply want a glimpse of what "higher" math is all about, you are urged to include Math 101 early in your curriculum. Math 101 can be taken concurrently with either Math 21a or 21b. This course is only offered in the fall. If you have had some experience with rigorous proofs and want a different taste of "higher" math, you might consider Math 152 in the fall. Neither Math 101 nor Math 152 is appropriate for people from Math 25, Math 55 or (with rare exceptions) Math 23.

- Math 23, 25 and 55 are the three introductory courses for students with strong math interests. They are geared towards new students. Math 25 and 55 are much more intensive than Math 23, but require much more out of class time. Students who don't wish to make the time commitment will do well to choose Math 23. Meanwhile Math 55 should be taken only by students with *extensive college level math backgrounds*. Each year several first-year students ask to skip the Math 25/55 level and start with Math 122 or another 100-level course. The Department, based on many years of experience, *strongly discourages* this. Even if you have taken several years of math at another university, even if you have seen every topic to be covered in Math 25 or 55, you will not be bored in these accelerated courses. The topics covered in Math 25 and 55 are not as important as the level and the depth of mathematical maturity at which they are taught. Taking Math 25 or 55 is the most intense mathematical experience you are going to have in any Harvard course, shared with the most talented of your peers. You may learn more advanced material in other 100- and 200-level courses, but never with the same speed and depth as in Math 25 or 55. These courses are not taught in any other university because no other university has the same caliber of first-year mathematicians. And

the courses are simply a lot of fun. Many students who have skipped 25 and 55 have been dissatisfied with their decision. In any event, you *must* speak with the Director of Undergraduate Studies if you plan to skip the Math 21-55 level.

- Math 112 and Math 121 are courses suitable for students from Math 21, and they provide an alternative entry-point for the department's more advanced courses in Analysis and Algebra respectively. They should not be normally be taken by students who have been through Math 23 or 25. If you are a sophomore and have taken Math 21 but are not yet comfortable with writing proofs, then consider including these courses in your plan of study.

Key courses at the 100 level

If you have taken Math 23, 25 or 55, or if you have taken Math 21 and gained some experience in writing proofs through courses such as Math 101, 112 and 121, then you are ready to take some of the courses at the 100-level that form the core of the Mathematics curriculum. Most of the courses at this level can be classified as belonging to one of the three main streams of mathematics: "Analysis", "Algebra" and "Geometry and Topology". Courses belonging to these areas are numbered in the ranges 110–119, 120–129 and 130–139 respectively. In each stream, there are two courses which are regarded as "core" courses, making a total of six central courses. These are:

- Math 113. Analysis I: Complex Function Theory
  Math 114. Analysis II: Measure, Integration and Banach Spaces
- Math 122. Algebra I: Theory of Groups and Vector Spaces
  Math 123. Algebra II: Theory of Rings and Fields
- Math 131. Topology I: Topological Spaces and the Fundamental Group
  Math 132. Topology II: Smooth manifolds

It is not necessary to include all six of these courses in your plan of study, but here are some points to bear in mind.

- Students from Math 55 will have covered in 55 the material of Math 122 and Math 113. If you have taken 55, you should look first at Math 114, Math 123 and the Math 131-132 sequence.
- With the exception just noted, you should consider including Math 122 early on in your curriculum. Algebra is a basic language of modern mathematics, and it is hard to comprehend advanced material without some familiarity with groups and related topics in algebra. The same remark applies to Math 123, to a lesser degree.

By the same token, Math 113 should also be taken early on as Complex Analysis is used in many other fields of mathematics. You will also find the topology you learn in Math 131 useful in many other areas: amongst other things, it provides the mathematical language with which to discuss continuity and limits in wide generality.

- Math 123 cannot be taken before Math 122; but in the other two streams, the courses can be taken in either order. Thus, Math 114 can be taken before or after Math 113, and the same applies to Math 131 and 132.

- You should try to fulfill the *distribution requirement* (i.e., the requirement to take at least one course in analysis, algebra, and geometry) early in your academic career. By your junior or senior year, you should be exposed to the main branches of mathematics; then you can choose the department's advanced courses. In any case, most 200-level courses assume (at least informally) familiarity with the basic tools of analysis, algebra, and topology.

Other courses at the 100 level

At this level, there are many other courses to choose from: Number theory in Math 124 or Math 129, Differential Geometry in Math 136, Probability in Math 154, Logic and Set Theory in Math 141 and Math 143, amongst others.

- It is a good idea to take a tutorial (Math 99r) during the sophomore or junior year. Many students found the tutorial to be one of the best courses they took at Harvard. Tutorials generally satisfy the Math Expository requirement and often lead to senior thesis topics. More about tutorials appears below.

- Students wishing to take a rigorous course in mathematical logic in years when Math 141, 142, 143, or 144 are not offered at Harvard should consider taking logic courses at M.I.T. In any event, the Harvard courses offer a good introduction to model theory, set theory and recursion theory — the three main branches of Mathematical Logic. Students interested in the more philosophical aspects of logic and/or in proof or set theory may want to take Philosophy 143, and those interested in mathematics of computation should look into Computer Science 121 and some of the other theoretical CS courses.

- Students interested in Combinatorics should look at Math 155, and may also want to look up M.I.T.'s listings in that area. If you want M.I.T. courses to count for the concentration credit, you must get permission in advance from the Director of Undergraduate Studies, Prof. Jacob Lurie (lurie@math).

- Students are encouraged to take courses from a variety of professors in the department and not just to "follow" one teacher. It is advisable to be exposed to different views and styles of doing mathematics.

200-level courses

100, 200 – What's the Difference?

The difference between 100-level and 200-level courses is fairly easy to summarize: 100-level courses are designed for *undergraduates,* whereas the 200-level courses are generally designed for *graduate* students. As far as course material goes, the 100-level courses are designed to offer a comprehensive view of all the major fields in pure mathematics. They emphasize the classical examples and problems that started each field going and they all lead to one of the fundamental results that motivates the further development of the field. In contrast, a 200-level course will assume you understand the basic ideas of a field. A 200-level course will set out the systematic, abstract foundations for a field and develop tools needed to get to the present frontiers.

The 100-level courses give you a good overview of mathematics, they foster intellectual growth, and they prepare you for your chosen career. This is not true of 200-level courses. These courses assume that you are interested in the subject, and that you are already fairly certain of becoming an academic mathematician. The amount you learn in such a course is often also entirely up to you. Your prerequisites, though correct according to the course catalog, may be entirely inadequate. Many courses are paired into 100-level and 200-level sequences:

Corresponding 100-level, 200-level Courses

Math 114	-¿	Math 212a,b	(Real Analysis)
Math 113	-¿	Math 213a,b	(Complex Analysis)
Math 122/123	-¿	Math 221	(Algebra)
Math 129	-¿	Math 223a,b	(Algebraic Number Theory)
Math 131	-¿	Math 231a,b	(Algebraic Topology)
Math 132/136	-¿	Math 230a,b	(Differential Geometry)
Math 137	-¿	Math 232a,b	(Algebraic Geometry)

Other 200-level courses are harder to classify, but cover topics equally central to modern mathematics. For example, Math 222 is a course on Lie Groups and Lie Algebras that draws on background material from Analysis, Algebra and Geometry.

Skipping 100-level Precursors

Students are *strongly* discouraged from taking any 200-level course before taking its 100-level precursors. Although it is possible in principle to learn a general abstract topic on the basis of the logic of its definitions and theorems alone, it is almost impossible to appreciate their significance and "feel" without studying the more down-to-earth background which led to them. *Moreover,* students are well advised to take basic classes in algebra, topology, and analysis before exploring the graduate curriculum: often a basic familiarity with other areas will be an assumed prerequisite. Certainly, it can't hurt. However, even this *may* not suffice.

Some graduate courses (notably 212a, 221a, 231a) often conform better to undergraduate expectations (set material, careful pace, motivation); the best way to tell whether this is going to happen is to go to the class yourself and find out. Beware, though: often these courses start in a user-friendly way (presenting simple definitions, for example), then speed up tremendously as time goes on.

Why Take 200-level Courses?

The reasons for *not* taking 200-level courses are legion. However,there are some equally good reasons for taking them. You will be treated like a graduate student, which is good if you want to be treated like one. There isn't much review of topics you may have already covered, requirements are fairly minimal, and, most importantly, you can learn a lot of substantial mathematics. (If this is what you want, tutorials are another good option. While they are undergraduate courses, one generally learns graduate material in them.)

A student who is considering graduate school in mathematics may want to include at least one 200-level course in his or her program (and, likewise, write a senior thesis) to get a taste of the likes of graduate school.

3 Other types of course

Tutorials

Tutorials are not required, but many students take a tutorial during their sophomore or junior year. Typically two tutorials are offered every semester.

Tutorials (Math 99r) are generally directed by graduate students, and have four to eight students in them. They tend to be less formal and structured than regular courses, yet require more involvement on the part of the students – students have to make presentations and write papers. Very frequently a topic studied in a tutorial leads naturally to a senior thesis. And the paper written for the tutorial generally satisfies the Math expository requirement.

The department places a description of the fall tutorials into concentrators' registration envelopes in September; a description of the spring tutorials is e-mailed to the concentrators e-mail list in January. Descriptions also appear during the first week of that semester on the undergraduate bulletin boards (one opposite room 320, and one near room 503 in the Math Department). The descriptions also appear on the Math Department's website at http://www.math.harvard.edu/. Often, tutorials get previewed at Math Table meetings. A special organizational meeting for tutorials is held in the first week of the fall semester. The spring semester tutorials are organized in the first week of that semester; see the Undergraduate bulletin boards for announcements.

Ordinarily only one Math 99r can count towards the concentration requirements.

All questions regarding tutorials may be addressed to the Director of Undergraduate Studies or the Undergraduate Program Coordinator, Cindy Jimenez (cindy@math).

Reading Courses (60r and 91r)

Honors candidates in their Senior year can choose to enroll in Math 60r to allow more time for thesis work. You can take Math 60r in the fall and/or spring semester. Math 60r is SAT/UNS only and does not count for concentration requirements. A student taking Math 60r in the fall must submit a one-page plan of thesis (including at least a preliminary bibliography) to Cindy Jimenez (rm. 334) **by 4 pm** of the last day of the fall reading period in order to pass.

If you want to learn a particular topic not covered in a regular course or a tutorial, you may consider taking Math 91r. For this you must find a faculty member willing to supervise your reading, as well as secure approval from the Director of Undergraduate Studies. Make sure that you, your supervisor, and the Director of Undergraduate Studies clearly agree on the topic, structure, frequency of meetings, and the grade requirements before you sign up for 91r. You should know exactly what is expected of you and how much guidance to anticipate. Ordinarily, Math 91r will not count for concentration requirements.

Note that Math 60r, 91r, and 99r require the signature of the Director of Undergraduate Studies on your study card.

Cross-registration at M.I.T.

Students may cross-register to take a course at M.I.T. This may be a useful option in years when a particular course is not offered at Harvard. Logic and Combinatorics offerings at M.I.T. have proven especially popular with Harvard students. Generally, classes at M.I.T. start a week before Harvard's in the fall, and contemporaneously with Harvard's in the spring. You may get concentration credit for M.I.T. courses, but consult

the Director of Undergraduate Studies *before* registering. Cross-registration petitions can be obtained at the Registrar's office or from your House's Senior Tutor.

If you are taking an M.I.T. course, you don't have to walk all the way down Mass. Ave. or even pay for the bus to get to class: you can use the Harvard Medical Area (M2) shuttle bus, which runs from Quincy Square (in front of Lamont) straight to M.I.T.

Related fields

Keep in mind that the concentration requirements for Mathematics require twelve half-courses, but only eight of those need to be listed under "Mathematics" in the Course Catalog. You are encouraged to round out your studies by including courses listed as "Related Fields" in the mathematics section of the *Handbook for Students*.

4 Sample Programs

The programs listed below should not be followed literally – they may not be balanced in workload between the fall and the spring semesters, nor are all the courses listed necessarily offered every year. They are examples designed to demonstrate the range of possibilities. You should determine your own program in consultation with your math faculty advisor or the Director of Undergraduate Studies.

(a) If you start with Math 1 a/b in your first year, you can continue with Math 21 a/b as a sophomore. Students who start with Math 1b in the fall of their first year normally take Math 21a in the spring; some choose also to take Math 21b concurrently with 21a in order to get "in sync." Some students who start with Math 1 a/b sequence freshman year and do extremely well may choose to take Math 23 or 25 their sophomore year, instead of Math 21 (if you are considering doing this, you should talk to Jacob Lurie, the Director of Undergraduate Studies (room 514)). Otherwise, you'll get a first feel for proofs and abstraction by taking Math 101, 112 or 121. A possible schedule is:

FR	**SO**	**JR**	**SR**
Math 1a	Math 21a	Math 112	Math 113
Math 1b	Math 21b	Math 121	Math 131
CS 50	CS 51	Physics 15a	Math 122
	Math 101	Stat 110	Phil 144

(b) Students who start with 21 a/b in their first year can take 101 either concurrently with Math 21 or in their sophomore year along with with 112, 122 and 131. Students are also encouraged to take Physics 15 a/b/c or Computer Science 51 to broaden their understanding of how mathematics applies to other disciplines. Students who wish to

write a senior thesis often take reading course or a 200-level course in the field of their senior thesis during their senior year.

FR	**SO**	**JR**	**SR**
Math 21a	Math 122	Math 141	Math 231a
Physics 15a	Math 131	Math 124	Math 114
Math 21b	Comp Sci 51	Math 99r	Math 231b
Math 101	Math 112	Math 132	Ec 2052

(c) A student with a strong interest in mathematics, or a strong mathematical background would most likely start with Math 23, 25 or 55 during the first year. His or her sample program might look like this:

FR	**SO**	**JR**	**SR**
Math 23a or 25a	Math 122	Math 114	Math 60r
Physics 15a	Math 131	Math 99r	Math 212a
Math 23b or 25b	Math 123	Math 129	Math 222
Physics 15b	Math 113	Math 132	Math 137

(d) Consider a student with a strong interest in mathematical physics, concentrating in Mathematics as a primary field and Physics as the secondary one, and who started with a Math 23a,b or Math 25a,b sequence and the Physics 16, 15b, 15c sequence. Some of the 100-level math courses of particular interests to physicists are Math 115, 132 and 136. The sequel to Physics 15c is Physics 143a,b. Choosing some of the math and physics courses with the most conceptual interaction, you might come up with the following to fulfill the math portion of the requirements. (You should consult with the Physics Head Tutor to plan the physics portion.)

FR	**SO**	**JR**	**SR**
Math 23a or 25a	Math 131	Math 115	Math 230a
Physics 16	Math 122	Math 132 or 136	Math 99r
Math 23b or 25b	Phys 15c	Physics 143b	Math 230b
Physics 15b	Phys 143a	Phys 181	Math 123

(e) A primary/secondary honors major in Mathematics and Computer Science is common. If you choose this option, you are required to write a thesis which applies ideas of computer science to a topic in pure mathematics, or vice versa. Mathematics courses of particular value here would be Math 141 (introduction to mathematical logic), Math 142 (Recursion Theory), Math 124 (number theory including primality tests and applications to codes), Math 130 (on axiomatic foundations of geometry), and Applied Math 107 (combinatorics). A possible program when Math is the primary which fulfills the

Math requirements is given below. Consult with the Computer Science Head Tutor to plan that portion of your course work.

FR	**SO**	**JR**	**SR**
Math 21a	Math 121	Math 122	Math 141
Math 101	CS 121	CS 207 or 226r	Math 191
Math 21b	Math 112	AM 107	Math 142
CS 51	CS 124	Math 130	Math 124 or AM 111 or Math 152

Math 99r Tutorial topics

(2015-2016)

Fall Tutorial 2015

Category Theory

Description: Category theory is central to the study of modern mathematics. Most of the subjects which have been profoundly influenced are, however, rather technical and typically lie outside of the standard undergraduate curriculum. On the other hand, insights gained from a categorical point of view can be very useful when learning mathematics at any level. We will introduce the subject by building on many familiar examples from basic algebra and topology. We will explain the meaning of categories, functors, natural transformations, limits, colimits, Yoneda's lemma, representability, and adjunctions. In order to make our exposition more meaningful and enjoyable, each definition will be followed by an explicit example in a category that the students are acquainted with. Once the basis of the subject has been throughly developed, we will venture into more advanced topics. Potential topics may include, but are not limited to:

(1) Abelian categories and homological algebra,
(2) Model categories,
(3) Higher categories.

Students' backgrounds and interests will be the main parameters taken into consideration when choosing topics.

Prerequisites: Math 122 and Math 131.

Contact: Danny Shi (dannyshi@math.harvard.edu)

Spring Tutorial 2016

Morse Theory

Description: Morse theory enables one to study the topology of smooth manifolds by studying smooth functions. According to Marston Morse, a "generic" smooth function can reflect the topology of the underlying manifold quite directly.

By definition, a Morse function on a smooth manifold is a smooth function whose Hessians are non-degenerate at critical points. One can proof that any function can be perturbed to a Morse function. The classical approach of Morse theory is to construct a

cellular decomposition of the manifold from the Morse function and then study the topology of the manifold accordingly. Each critical point of the Morse function corresponds to a cell with dimension equal to the number of negative eigenvalues of the Hessian matrix.

However, the classical method cannot be generalized to infinite dimensional cases because attaching a cell of infinite dimension does not change the homotopy type. An approach for the infinite dimensional case was introduced by Andreas Floer: instead of studying the cellular decomposition, he considered the flow lines of the Morse function and defined homology groups out of it. If the dimension of the manifold is finite, such groups are canonically isomorphic to the singular homology groups. In the infinite dimensional case, this turns out to be a new invariant and is now called "Floer homology".

In the tutorial, we are going to introduce both the classical and Floer's approach of Morse theory on finite dimensional manifolds. Then, as an application, we will talk about the smooth h-cobodism theorem and give a proof of the generalized Poincare conjecture in higher dimensions. If time allows, we will talk about other applications of Morse theory such as the Lefschetz hyperplane theorem and the existence of closed geodesics on compact Riemannian manifolds.

Prerequisites: Students need to be familiar with the concepts of smooth manifolds, tangent and cotangent bundles, and tangent maps. Some knowledge in homology theory would be helpful but is not required.

Contact: Boyu Zhang (bzhang@math.harvard.edu)

Partitions, Young Diagrams and Beyond

Quotation: The theory of partitions is one of the very few branches of mathematics that can be appreciated by anyone who is endowed with little more than a lively interest in the subject. Its applications are found wherever discrete objects are to be counted or classified, whether in the molecular and the atomic studies of matter, in the theory of numbers, or in combinatorial problems from all sources. Gian-Carlo Rota

Description: A partition of an integer n is a finite weakly decreasing sequence of positive integers with a sum equal to n. It can be visualizedas a Young diagram: a collection of cells arranged in left-justified rows with row lengths given by elements of the sequence.

Despite such elementary description, Young diagrams occur in a variety of interplays between combinatorial and algebraic structures, related in particular to group representation theory, algebra of symmetric polynomials, and beautiful identities with series. They lead to surprising connections, in particular the knowledge about Young diagrams and representations of the symmetric group sheds light on questions such as:

How many times do you need to shuffle a deck of cards to make it close to random? Or: What can we say about the length of a longest increasing subsequence of a random permutation? Such interplays and their consequences will be the focus of this tutorial.

Prerequisites: Only basic linear algebra and interest in combinatorics. All other notions will be introduced during the course.

Contact: Konstantin Maveev (kmatveev@math.harvard.edu)

Honors in Mathematics
Writing a Senior Thesis
(2015-2016)

Candidacy for Honors

To receive honors in Mathematics, a student *must submit a senior thesis*. This is the only requirement for honors eligibility, beyond basic requirements of the Mathematics concentration. The Mathematics Department recommends students for graduation with honors in Mathematics on the basis of course grades in mathematics, the senior thesis, and a thesis examination. The final award of "latin" honors (*cum laude*, *magna cum laude* or *summa cum laude*) is made by the Faculty of Harvard University in accord with rules set forth in *Handbook for Students*, based on the Department's recommendation and a students grades and subject to overall percentage quotas.

Harvard University also awards degrees *cum laude* "on the basis of a student's overall record": that is, on the basis of grades in all subjects. There is no thesis requirement for this version of the *cum laude* degree. Under present faculty rules, students who qualify automatically receive their degrees *cum laude* if they do not receive honors in a field.

You should consult the *Handbook for Students* for a more detailed description of the general regulations relating to honors.

What is an Honors Thesis in Mathematics?

An honors thesis in Mathematics is an original presentation of an area or subject in pure or applied mathematics. A typical thesis is an original synthesis of knowledge culled from a number of sources in the published literature. A thesis can contain substantive, original mathematics, but most do not.

Why Write a Thesis?

The Mathematics Department strongly recommends that its concentrators write a senior thesis. Writing a thesis provides a glimpse of life as a graduate student in mathematics, and as a professional mathematician. It will also propel you towards the frontiers of current mathematical research. Moreover, working on your thesis can be an opportunity to interact closely with a working mathematician (your thesis advisor), which by itself, offers intellectual rewards.

In the past, almost all the seniors who wrote theses felt that working on one was the most challenging, confidence-raising, and fulfilling experience in their undergraduate careers.

Choice of Thesis Topic

Any subject which makes genuine use of mathematics at the college level is suitable for a senior thesis. The topic may be in pure or applied mathematics, subject to approval from the Director of Undergraduate Studies. In general, a thesis on a rather narrow subject works well, while one presenting the elements of a large theory is less satisfactory

both mathematically and in its value to the student. In judging theses more weight is given to the quality of the presentation than to the sophistication of the topic.

Start thinking about possible topics for a thesis early on. Students planning to write an honors thesis are advised to give serious thought to the choice of topic during their junior year. Your courses, your advisor, as well as Math Table lectures, may serve as sources of questions or subjects that interest you. In the past, tutorials and reading courses have proved especially useful in generating ideas leading to a good senior thesis.

It is important to choose a topic which interests you. Without a real interest it is difficult to do the amount of work necessary to write a good thesis. On the other hand, some students search too long for a topic that they will "fall in love" with, ending up with very little time to actually research and write the thesis. (You should merely like a topic or a field of your thesis. Often you simply know too little about the topic at the outset to realistically decide whether it is what you *really* want to study or not. Just don't pick a topic that bores you.) All things being equal, it is best to write a thesis in an area where you have taken several courses or a tutorial already, and therefore, have some perspective.

Thesis Advisor

If you have not chosen a topic on your own before the end of your junior year you should seek the advice of several faculty members as to areas of mathematics suitable for your senior thesis. Math Table is a good source of informal information too. Spend the summer before your senior year reading up in these fields.

At the end of your junior year or at the very start of your senior year, approach a faculty member whose field of expertise covers your chosen topic and consult him/her regarding your plans. Your academic advisor (the person who signs your study card) is usually not your thesis advisor; but he or she may direct you to a faculty member who would be better suited to be your thesis advisor.

All senior theses are written with the explicit advice of a faculty member. A common pattern is to take a reading course (Math 60r, SAT/UNS) during the fall term of your senior year and continue consulting with the same faculty member informally (or again in the context of Math 60r) during the writing of your thesis in the second semester. Note that your thesis advisor can help the department evaluate your thesis when honors decisions are made.

It may be possible to arrange for a professor from MIT or from another department at Harvard to serve as your thesis advisor. However, in such cases the student must have a second advisor in our department, and must get prior approval from the Director of Undergraduate Studies. In the past a few students who were advised solely by faculty from another department ended up with weak theses and did not receive an honors degree. Therefore, any student whose primary advisor is not a member of the Harvard Math Department is strongly advised to submit to the Director of Undergraduate Studies a complete draft of their thesis 30 days before the final due date for senior theses. This will give the departmental advisor the chance to suggest modifications that can put the thesis into compliance with the department's standards.

As soon as you have decided upon your thesis advisor and the topic, notify the Undergraduate Studies Coordinator, Cindy Jimenez (room 334, e-mail cindy@math).

If you have no topic or no advisor by late September of your senior year, please see the Director of Undergraduate Studies, Jacob Lurie (lurie@math), for help.

Getting Help

Choosing a thesis advisor and a topic can be a nerve-racking experience. Furthermore, while working on your thesis you may encounter difficulties, both logistic and otherwise. (For example, some students have trouble defining their goals or circumscribing the thesis topic; some may even decide to change the topic and/or advisor in mid-semester.) In all these circumstances you are encouraged to talk with the Director of Undergraduate Studies, Jacob Lurie (lurie@math).

Enrolling in Math 60r

Seniors can free up time for thesis research by enrolling in Math 60r. You must get the signature of the Director of Undergraduate Studies and you must enroll SAT/UNS. You can enroll for fall, spring, or both semesters. Students enrolled in Math 60r in the fall need to submit a thesis plan to the Director of Undergraduate Studies before the end of the fall reading period (see below).

Plan of thesis

Each candidate for honors in mathematics would do well to submit a thesis plan, one or two pages long, including at least a preliminary bibliography by 4 pm of the last day of the reading period in December to Cindy (rm 334). If you are enrolled in Math 60r in the fall, the failure to submit such a thesis plan by the deadline will result in an unsatisfactory grade for Math 60r.

The Thesis

Originality

An honors thesis in mathematics is not expected to be (and very rarely is) an original contribution to mathematical research. Only originality of presentation is expected. You should study several presentations of your subject until it is thoroughly assimilated and then write your own presentation of the subject. Theses which are drawn from a single source are not acceptable.

Occasionally, students do make original contributions. These are, of course, welcome; but a student is ill-advised to start work on a senior thesis determined to solve some outstanding classical problem. Progress in research is usually made by acquiring a good grasp of existing knowledge and answering successive small questions. If you do discover something new, be sure to consult with your advisor or some other faculty member about it. He or she may be able to help you go further or protect you from the embarrassment of a serious mistake.

Format

No length or format is prescribed for senior thesis. However, theses exceeding thirty typewritten pages put a considerable strain on the staff and rarely get as much attention as they may deserve. Twenty to twenty-five pages (typewritten) might be considered average; certainly many shorter theses have been judged *summa* quality.

It is not necessary to have your senior thesis typed. A *legible* handwritten thesis is entirely acceptable. However, you may want to typeset your thesis using the math-oriented language LaTeX available on the FAS computer system.

A bibliography *must* be included with your thesis. Please do not forget to put your full name, e-mail address, telephone number, and your thesis advisor's name on the front page of the thesis.

Some old theses are available in Cindy Jimenez's office (rm. 334) for you to look at with regard to style, length and general format.

Deadline

Two copies of the thesis must be handed in to the Undergraduate Studies Coordinator, Cindy Jimenez, in room 334 *no later than 4 pm* on the first Monday after spring recess.

Students Receiving a March Degree or for whom the Fall Semester is their Final Semester

Two copies of the thesis are due by 4 pm on the Monday after Thanksgiving in the office of the Undergraduate Studies Coordinator Cindy Jimenez (rm. 334). Students who wish to plan a timetable for March degree thesis should consult with the Director of Undergraduate Studies, Jacob Lurie (lurie@math).

Hoopes Prize

Your advisor may wish to nominate your thesis to the Hoopes Committee for consideration for the Hoopes Prize, which carries substantial monetary rewards for you and your thesis advisor. If you think that your advisor likes your thesis but may not be aware of the Hoopes Prize, don't hesitate to mention to him or her this possibility. In the past, a large portion of theses submitted for this prize from this department have won it. However, be forewarned that even a super Math thesis will fail to win a Hoopes Prize if its introduction (at least) is not written so that a non-mathematician (but scientifically literate individual) can understand it.

The Thesis Examination

Two weeks after you submit your thesis you may inquire with Cindy Jimenez (rm. 334, Cindy@math) as to which faculty member has been nominated to be your thesis reader. You should then get in touch with your reader to arrange a mutually convenient time for the thesis examination (usually held early in the spring reading period).

Whether the thesis examination will be oral or written will be decided by the thesis reader. If written, the exam will generally be no more than two hours. The aim of the exam is to test whether you have really come to an understanding of your chosen topic. The examination is confined to questions concerning the thesis, direct applications of the thesis, and mathematics related to it. In particular, you are most strongly advised to keep the following very much in mind while writing your thesis and preparing for the thesis exam. No matter how impressively advanced your thesis topic, and no matter how well written your presentation, you will not get a highest honors recommendation if you don't fully understand your subject, or if your understanding is so narrow that you falter on

questions that go somewhat to the side of your chosen path through your subject area.

After your thesis has been read and your thesis examination corrected, you may, if you wish, make an appointment to discuss your thesis and the examination with the reader.

Honors Recommendations

Honors recommendations are voted by the Department at a meeting in mid- or late May. Theses and the results of thesis examinations have great weight in formulating these recommendations, but it cannot be precisely quantified; it isn't a matter of simply computing some sort of weighted average. The departmental recommendation may be no honors, honors, high honors, or highest honors. Sometimes the recommendation is conditional on grades in the courses you have just taken.

All candidates for honors are expected to maintain honors-level grades in their math (and related) courses and to have submitted for their senior thesis a clear presentation of material culled from several sources.

A solid, workmanlike presentation, free of substantive errors, will typically receive a recommendation of honors. Students recommended by the department for high honors are expected to have shown insight into the subject and meaningful originality of presentation in their senior thesis and the thesis examination. They are also expected to maintain a high average in their math (and related) courses. In addition to the above, students recommended for the highest honors are expected to have achieved the kind of mastery of the subject generally exhibited by working mathematicians. (They are not required to have proved original results).

Be forewarned that there is, of necessity, a reasonable degree of subjectivity in the Department's decision-making process. The difference between the highest and high honors recommendations is often intangible. For example, highest honors does not require a thesis with original theorems. It requires neither extra course work in Mathematics above the 12-half-course requirement, nor a record with graduate courses. On the other hand, neither a straight A average nor a thesis with original results will ensure a highest honors recommendation from the Mathematics Department. In any event, only a few highest honors recommendations are made in any given year.

Recommendations from the departments are translated by the Administrative Board into recommendation to the Faculty for degrees without honors, *cum laude*, *magna cum laude*, and *summa cum laude*, using criteria explained in the *Handbook for Students*. These recommendations are acted on by the faculty of Harvard University at a meeting preceding the commencement.

In practice, those recommended by the Department for honors or high honors almost always receive their degrees *cum laude* or *magna cum laude*, respectively, provided their grade-point average is above the relevant cut-offs set forth in the *Handbook for Students*. About two-thirds of those recommended for highest honors graduate *summa cum laude*; the remainder usually receive their degrees *magna cum laude* with highest honors in Mathematics.

Dates to Remember:

- **Sophomore and junior years** – Look for potential thesis topics.
- **End of junior year** – Approach several faculty members for specific suggestions of thesis topics.
- **Summer before senior year** – Read up in the fields of potential thesis topics. Try to make the decision on the topic and become familiar with basic literature on it.
- **September of the senior year** – Choose your thesis advisor. Notify the Undergraduate Studies Coordinator about your choice. Enroll in a reading course supervised by your advisor (optional). (See the Director of Undergraduate Studies, Jacob Lurie, if you have no topic or advisor by the end of September).
- **Last day of the Fall Reading Period** – Submit a thesis plan to the Undergraduate Studies Coordinator Cindy Jimenez by 4 pm.
- **January/February of the senior year** – Start actual writing!
- **March 1**$^{\text{st}}$ – Submit a thesis draft to your advisor for feedback and advice. Submit a draft to the Director of Undergraduate Studies also if your primary advisor is not a member of the Mathematics Department.
- **First Monday after spring recess** (or first Monday after Thanksgiving for students planning to receive a March degree or if the fall semester is their final semester) – Two copies of the thesis are due by 4 pm in the office of the Undergraduate Studies Coordinator Cindy Jimenez (rm. 334).
- **3rd Week of April** – Contact Cindy Jimenez (cindy@math) to find out the name of your thesis reader for arranging a thesis examination.

Graduate Schools and Fellowships in Mathematics

(2015-2016)

Choosing the Right Grad School

Your choice of a graduate school is a major step in your career as a mathematician. Below are some criteria you may consider for choosing the right school for you. Not all the criteria apply to everybody. We strongly urge you to seek advice from several faculty members familiar with you and/or your field of interest early in the fall semester of your senior year to try to sort out what may be the best "fit" between you and a graduate school.

Thesis Advisor

A thesis (dissertation) advisor plays a **very** important role in the student's graduate education. After all, a dissertation generally consists of making a new advance, solving an unsolved problem. And since the problem is not yet solved, the advisor, presumably, has no solution for it either. Therefore it takes a good mathematician, with sound intuition and deep insights to help his/her advisee choose a problem that will, on the one hand, not wind up being so hard that a student can't solve it or parts of it, nor, on the other hand, so easy that a student wouldn't get to learn a lot of beautiful mathematics on his/her way to solving the problem.

Most math departments do not have the resources to excel in all branches of mathematics. You should ascertain that a university you are planning to apply to has top-quality tenured faculty members pursuing research in your potential field of specialization. But you don't necessarily have to go to a leading grad school to get a good advisor. There are a number of mathematics departments in this country which may not be at the top of the pack overall, but which have one or more top-quality mathematicians who can be excellent thesis advisors. Your academic advisor at Harvard can help you sort out which departments are strong in which areas.

Breadth and Depth

As important as it is to choose a school with strong reputation in your field of interest, it is also important to balance this criterion with considerations about the overall breadth of the department. It is common for one's interests to change as learning progresses, and you want to leave yourself the freedom to switch fields or advisors without leaving your chosen graduate department.

The Strength of Your Peers

The quality of other graduate students in the program is also very important. During the first few years of graduate study you will learn much from other graduate students, so it is very helpful to have talented peers.

On the other hand, going to the most competitive school may not be the best decision for everybody. You can sometimes get more time and attention from faculty if you are at the top of a weaker pack, rather than near the bottom of a stronger one. Again, your academic advisor can help you decide what would be best balance in your case.

The Student-to-Faculty Ratio

An equally important factor is the student-to-faculty ratio. In some universities, it is not uncommon to have 10 to 15 grad students working under one advisor. Obviously under these conditions, some students won't get the attention they require.

Visiting the Schools

When making a final decision on graduate schools, **visit** all your serious choices. Talk to faculty members about their research interests and the number of dissertations they are advising. Ask where their former students are employed.

A good source of information are the graduate students currently enrolled in the program. Talk to them about accessibility and reputation of potential advisors in your field of interest. Find out what is the average number of years students take to complete their dissertation, and (very important) what is the dropout ratio. Look into the housing options (very rarely do grad students live in the dormitories for more than a year, if at all) and the teaching load of teaching assistants. Try to decide if you will be comfortable spending some number of years at that university or in that town.

Getting Advice

Finally, you should talk to your academic advisor, the Director of Undergraduate Studies, and other faculty you know well. They can evaluate your choices, give you the latest information about the departments you are considering, and help you decide what is best for you.

Applying to Harvard

The Harvard Math Department encourages its own undergraduates to go elsewhere for graduate study because it is a good idea for a student to get to know other mathematicians, to be exposed to alternative tastes and styles of doing mathematics.

The Admissions Process

Recommendations

An important criterion used by many admissions committees is the content of faculty recommendations. Graduate schools look closely at evidence not only of mathematical ability, but of motivation and tenacity. In contrast to college admissions, extracurricular

activities and non-academic character traits are not given much weight. Usually two or three recommendations are required, and you should be making the effort throughout your undergraduate career to let a few faculty members know you well (e.g., by dropping by their office hours, inviting them to the semi-annual faculty dinners held at your House).

If you are writing a thesis, you might ask your thesis advisor for a letter of recommendation. Working on your thesis during the summer and fall of your senior year with your thesis advisor provides an opportunity for him or her to get to know you better.

If you took a course from a professor and did relatively well, you may want to ask him or her to write a letter of recommendation right away. He or she may not remember all the details of your performance a year or two later. The recommendation can later be fine tuned to address specific graduate schools or fellowships. If you are thinking of taking some time off after college and applying later, you may still want to get your recommendations written while you are in school. Remember that faculty members often take sabbaticals, change universities, etc. Ask your recommenders to send copies of your recommendations to your undergraduate house to be included in your personal folder; they may be useful later on. To place a copy of your recommendation in your undergraduate house file you need to obtain a form from your House's Senior Tutor. Harvard will keep these files indefinitely, and will mail the letters you want to graduate schools or employers at your request.

Essays

You will be required to write one or two application essays. Typically you will need to describe your academic background, your achievements to date, what experiences led you to want to get a Ph.D. in math, and what areas of research interest you most. Those essays give you an opportunity to explain away some bad grades you may have had, to demonstrate your new-found love of math and resolve to succeed, and convince the admissions committee that you not only have the requisite intellect, but that you are going to persevere through some tough moments in your career and finish your dissertation. They want to make sure that you aren't going to grad school just because you could think of nothing better to do, or because you missed the LSAT deadline. You don't have to feel that you should know your dissertation topic by your senior year in college. You should merely demonstrate that you really are interested in mathematics and are relatively focused and determined.

Grades and Test Scores

Besides recommendations and essays, other criteria for admissions include grades and scores on the Graduate Record Examination (GRE). Most selective math departments do not put too much weight on the differences between good and great scores, but having poor grades in math courses or poor GRE scores can hurt your chances of admissions. Many schools look at your transcript to see evidence of substantial exposure to serious mathematics (e.g. some graduate level courses) and are quite understanding about some poor grades.

Taking the GRE

Most universities require applicants to take two parts of the GRE — the general and the subject tests. The general part is similar to the SAT. The questions on the subject test in Mathematics may be quite different from the math most students learn at Harvard, and you are well advised to look over the material tested ahead of time. Even if many of the problems may seem easy, you have less than a minute per question, and if you have to derive everything, you won't finish the test.

GRE, at least in the usual "paper and pen" format, is offered only 3 times a year: in October, December and April. Although you *can* take both the general and the subject part on the same day, most students prefer not to. Therefore many take at least the general part of the GRE during their junior year. In any case, it's a good idea to get the GRE out of the way soon. When you start working on your thesis and filling out applications for grad schools and fellowships, worrying about the GRE will be the last thing you will want to do.

Also keep in mind that you have to register to take the GRE more than a month in advance; if you want to take the test at a place somewhere near Cambridge, you should get moving several months before the test date. For example, if you plan to take GRE in October, and would prefer to take it in Boston rather than, say, in Swampscott (remember, you have to be there at 8am) you should register as early as July. GRE information pamphlets are available from OCS, from the GSAS admissions office at Byerly Hall. You can also obtain them from ETS web page, http://www.ets.org/index.html or from http://www.gre.org.

Note that if you apply for National Science Foundartion (NSF) Graduate Fellowships, then the NSF will actually pay for your GRE test – provided you take it in December. See the NSF application for more information.

Deadlines

The deadlines for graduate school applications range from early December to early January. Most schools usually require you to complete your application folder by January 1st or 15th. The deadlines for fellowship applications start as early as October.

Fellowships and Financial Aid

Unlike what many people think, **you don't need to pay to go to graduate school in mathematics.** Grad students are usually paid (albeit not much) to study. There are several ways post-graduate education in mathematics is financed.

National Fellowships

A few students are able to win national fellowships. The national fellowships are awarded by various government and private foundations, and some pay rather large stipends plus

tuition for the first three to five years of graduate school. Currently, we are aware of support being offered by the following organizations:

- The National Science Foundation, NSF, is the largest funding agency for graduate work in mathematics, offering both regular Graduate Fellowships and some special ones for minorities. The deadline for submitting the first part of the application is usually in November. NSF will even pay for you to take your GRE's in December!

  The NSF web page is http://www.nsf.gov or go directly to *fastlane*, the electronic fellowship application submission web page, http://www.fastlane.nsf.gov.

- The Fannie and John Hertz Foundation, a private foundation that purports to support only students in "applied physical sciences", but, in reality, often funds study for 5 or more years in many areas of pure mathematics. It also has one of the most lucrative stipends. Their web site is at http://www.hertzfndn.org.

- The Department of Defense. It funds the National Defense Science and Engineering Graduate Fellowships (NDSEG). This is a 3-year fellowship, and it is *not* similar to ROTC — you don't have to promise to serve in the military or have any other special obligations to the government if you win a fellowship. To get the application and information, check their web site at http://ndseg.asee.org. The application deadline is usually the first week of January.

- Canadian students should look to The Natural Sciences and Engineering Research Council (NSERC), which provides scholarships for graduate study. For information check their web site at http://www.nserc-crsng.gc.ca. (NOTE: the deadlines for these lie early in the fall semester!)

Flyers put out by some of these fellowships are posted on the undergraduate bulletin boards–one is opposite room 320 and another is opposite room 503.

The Office of Career Services (OCS) Web site offers useful information for applying to graduate school and for finding sources of funding. Please review the Fellowships section on the OCS Web site at http://www.ocs.fas.harvard.edu/students/fellowships.htm. Also, OCS has a library with information on fellowships, and its staff can help you enormously in your pursuit.

The Harvard Graduate School of Arts and Sciences (GSAS) offers fellowship information online at http://gsas.harvard.edu. The online publication "Financing Graduate Studies" is available to download, and the interactive database "Graduate Guide to Grants" will allow you to search for specific grants and provides contact information, application guidelines and restrictions, and deadlines.

Finally, check the Web sites of the universities where you plan to apply. Like GSAS, many graduate schools offer graduate funding options specific to their institutions.

Generally, undergraduates are urged to try their luck in all nationwide fellowships. Harvard students have been very successful in winning these awards. Some fellowships have early deadlines (as early as October) and some pay more attention to the GRE scores and

grades than university admissions offices.

University Fellowships

A university itself may also offer a number of fellowships for students who will accept its offers of admission. These can be as lucrative as the nation-wide fellowships, and you generally don't need any separate application – your application for admission automatically enters you into the competition.

Teaching Assistantships

Students who do not get fellowships usually receive teaching assistantships. Those generally carry a tuition waiver and a stipend that is sufficient for living expenses in exchange for teaching, grading, or assisting in low-level math courses. (Many departments won't let first-year students teach. These often pay incoming students a stipend, and have them begin teaching in their second year). Some students can get research assistantships which let them stop teaching and concentrate exclusively on research.

Masters Degrees

Few students enter graduate school in mathematics for an M.A. However, if you think that this is something you may want to do, you should discuss your situation with your academic advisor or the Director of Undergraduate Studies. In many schools, masters degrees are awarded only on the route to a Ph.D.; also, students who drop out from a Ph.D. program after a year or two are often able to get the masters degree on their way out (you should check the policies of individual departments). If you are applying for a masters program only, you may have less (in some universities, significantly less) chance of getting financial aid. Some universities do not admit students who want to get a terminal M.A. degree.

Harvard's A.B.–A.M. Degree Program

Harvard students with Advanced Standing may wish to apply for the A.B.–A.M. degree. These students must meet both the academic and course requirements for each of these two degrees. A given course can be counted for only one of the two degrees, i.e., one course cannot meet the requirements for the A.B. degree and then be counted again for the A.M. degree. In addition to the course requirements, any candidate for the A.M. degree in Mathematics at Harvard *must take a special language exam* to demonstrate the ability to read mathematics in either French, German or Russian. These tests are administered by the department only once a semester. For more information on the language exams please contact the Graduate Program Coordinator. Any undergraduate who wishes to apply for the A.B.–A.M. degree must file an application form for the graduate program in mathematics just as any other student files for graduate work at Harvard. For information on the degree, contact the office of Advanced Standing.

Study Abroad

There are many fellowships and scholarships for study abroad. Information about them can be obtained on the Office of Career Services (OCS) website, under "Global Opportunities", or from your House Fellowships Tutor. Particularly useful are the *Guide to Grants* and the *Guide to Study Abroad*, published annually by OCS. Some of the math/science oriented fellowships are the:

- **Churchill Scholarship** (for study at Cambridge University),
- **Herschel Smith Harvard Scholarship** (also Cambridge),
- **Weizmann Institute of Science Scholarship** (the Weizmann Institute in Rehovot, Israel).

In the past, math majors from Harvard have also been particularly successful in competing for the **Marshall Scholarships**, and for the **Fulbright Grants**, especially for study in Israel (it seems that at least one grant a year almost always goes for a math student to study Logic at the Hebrew University of Jerusalem, and Israel is one of the very few countries where one does not have to know the native language in order to receive a Fulbright grant to study there). Application deadlines for some of the these fellowships start as early as September.

Further Information and Advice

For further information and advise about graduate programs and fellowships in mathematics, please talk to the Director of Undergraduate Studies, Professor Jacob Lurie, your faculty advisor or the House Fellowship Advisor. They can greatly assist you in determining what recommendations and information outlined here applies in your case, and may help you get more information. And watch for advertisements from fellowships and graduate programs on the undergraduate bulletin boards.